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AN ALTERNATIVE TO THE SEA COUPLING LOSS FACTOR: RATIONALE AND METHOD FOR EXPERIMENTAL DETERMINATION

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For a combination of theoretical and phenomenological reasons, it is argued that the conventional SEA coupling loss factor is not the most appropriate form of coefficient for relating energy transfer between SEA subsystems to their vibrational states. An alternative 'power transfer coefficient' is suggested and an experimental method for its determination, which obviates the need to measure input powers, is proposed.

1. INTRODUCTION

The coupling loss factor conventionally employed in Statistical Energy Analysis is defined by analogy with the dissipation loss factor, thus: time-average power dissipated by subsystem $i = \eta_i \omega E_i$; time-average power transferred by subsystem i to subsystem $j = \eta_{ij} \omega E_i$; time-average power transferred by subsystem j to subsystem $i = \eta_{ji} \omega E_j$. Here E_i represents the total time-average energy stored in subsystem i in time-stationary vibration and ω is the centre frequency of an analysis band of width $\Delta\omega$.

This definition produces a non-symmetric loss factor matrix

$$\omega \begin{bmatrix} \sum_{i \neq 1} \eta_{1i} + \eta_1 & -\eta_{21} & \cdot & -\eta_{k1} \\ -\eta_1 & \sum_{i \neq 2} \eta_{2i} + \eta_2 & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ -\eta_{1k} & \cdot & \cdot & \sum_{i \neq k} \eta_{ki} + \eta_k \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_k \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_k \end{bmatrix}, \quad (1)$$

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in which P_i represents time-average input power to subsystem i. Introduction of the so-called SEA reciprocity relation, $\eta_{12}n_1 = \eta_{21}n_2$, allows the matrix to be written in a symmetric form as

$$\omega \begin{bmatrix} n_1\left(\sum_{i\neq 1}\eta_{1i}+\eta_1\right) & -n_2\eta_{21} & \cdot & -n_k\eta_{k1} \\ -n_1\eta_{12} & n_2\left(\sum_{i\neq 2}\eta_{22}+\eta_2\right) & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ -n_1\eta_{1k} & \cdot & \cdot & n_k\left(\sum_{i\neq k}\eta_{k1}+\eta_k\right) \end{bmatrix} \times \begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ \cdot \\ \cdot \\ \cdot \\ E_k/n_k \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_k \end{bmatrix} \quad (2)$$

where n denotes modal density, and E_i/n_i is the average energy per mode (modal energy) of subsystem i times the analysis bandwidth $\Delta\omega$.

2. AN ALTERNATIVE DEFINITION

On the basis of equation (2) alone one can make out a case for defining a new set of coefficients linking transferred and stored energy, namely

$$M_{ij} = \eta_{ij}\omega n_i = \eta_{ji}\omega n_j = M_{ji}. \quad (3)$$

These have previously been termed 'power transfer coefficients' [1] and 'modal coupling factors' [2]. The analogous 'power dissipation coefficients' are $M_i = \eta_i\omega n_i$. The latter are physically significant since they represent the degree of modal overlap of the uncoupled modes of each subsystem. The modal overlap factor has great significance in SEA because it influences ensemble response statistics and also features in a recently developed indicator of strength-of-coupling between subsystems [3, 4]. If $M_{ij}/(M_iM_j)^{1/2}$ is sufficiently large, the coupling may be said to be strong and the actual value of M_{ij} then falls below the value corresponding to the travelling wave power transmission coefficient which is conventionally employed in SEA analysis.

A justification for the use of M_{ij} rather than η_{ij} and η_{ji} is that the former depends only upon the linear dimensions of the interface between subsystems i and j, whereas the coupling loss factors depend, in addition, upon the other dimensions of the subsystems, as shown by the comparison in Table 1, in which $\tau_{\alpha\beta}$ is the diffuse field vibrational wave

power transmission coefficient of the interface and c is the wave group speed of subsystem α.

The power transmitted through an interface is proportional to the product of the physical extent of the interface and the intensity of vibrational waves approaching the interface. The intensity is equal to the product of the energy density of the waves and their group velocity. The energy density is proportional to the modal energy [1]. It is therefore illogical to relate the transmitted power to the *total* energy of a subsystem since, for a given value of energy density, the total energy varies in proportion to the spatial extent (length, area or volume) of the subsystem, whereas the transmitted power is unaltered. The awkward consequence of employing the total energy is that the associated coupling loss factor depends upon all the linear dimensions of the subsystem, and not just on the relevant quantity, namely the extent of the interface, as shown by Table 1. One consequence of employing M_{ij} is that the so-called 'Smith criterion' of coupling strength may be expressed as $M_{ij} \ll M_i$ and M_j. (However, the general validity of the criterion has been questioned [3, 4].)

3. EXPERIMENTAL DETERMINATION OF POWER COEFFICIENTS

Although it may be argued that the use of power transfer and dissipation coefficients rather than coupling and dissipation loss factors is logical from a theoretical point of view, it appears to present difficulties in terms of application of the widely used power injection method for the experimental determination of coupling and dissipation power parameters, via inversion of the subsystem energy matrix. This is because the modal energies of structural subsystems cannot be directly determined from measured quantities, whereas the total energies can be estimated from vibration velocity response measurements, made at a number of sampling points distributed over the spatial extent of a subsystem, together with an estimate of the associated effective mass: $E = m_{ei}\langle v^2 \rangle_{\cdot}$ where the brackets indicate spatial average.

However, there is, in principle, a way to overcome this problem which brings with it the considerable advantage of avoiding the need to measure input power in the power injection method. The time-average input power from a harmonic point force acting on a distributed elastic subsystem is given in terms of the velocity response at the driving point by

$$P_{in} = \tfrac{1}{2} \operatorname{Re} \{\tilde{F}\tilde{v}^*\}, \tag{4}$$

TABLE 1

Power transfer coefficients and equivalent coupling loss factors of spatially uniform subsystems

	$M_{\alpha\beta}$	$\eta_{\alpha\beta}$
One-dimensional subsystems of lengths L_α and L_β	$\tau_{\alpha\beta}/2\pi$	$\tau_{\alpha\beta} c_{g\alpha}/2\omega L_\alpha$
Two-dimensional plane subsystems of areas S_α and S_β and interface length L	$(k_\alpha L)\tau_{\alpha\beta}/2\pi^2$	$\tau_{\alpha\beta} L c_{g\alpha}/\pi\omega S_\alpha$
Three-dimensional subsystems of volumes V_α and V_β and interface area S	$(k_\alpha^2 S)\tau_{\alpha\beta}/8\pi^2$	$\tau_{\alpha\beta} S c_{g\alpha}/4\omega V_\alpha$

in which the tilde indicates 'complex amplitude' and the asterisk indicates 'complex conjugate'. The force and velocity are related through the complex point mobility $\tilde{Y}$. In spectral terms

$$P_{in}(\omega) = S_{ff}(\omega)\,\mathrm{Re}\,\{\tilde{Y}(\omega)\} = S_{vv}(\omega)\,\mathrm{Re}\,\{1/\tilde{Y}(\omega)\} = [S_{vv}(\omega)/|\tilde{Y}(\omega)|^2]\,\mathrm{Re}\,\{\tilde{Y}(\omega)\}, \tag{5a–c}$$

where S_{ff} and S_{vv} are the autospectral densities of force and response velocity at the driving point.

The power input in a frequency band $\Delta\omega$ is therefore given by

$$\int_{\Delta\omega} P_{in}(\omega)\,\mathrm{d}\omega = \int_{\Delta\omega} S_{ff}(\omega)\,\mathrm{Re}\,\{\tilde{Y}(\omega)\}\,\mathrm{d}\omega = \int_{\Delta\omega} [S_{vv}(\omega)/|\tilde{Y}(\omega)|^2]\,\mathrm{Re}\,\{\tilde{Y}(\omega)\}\,\mathrm{d}\omega.$$

In many practical measurements, power is injected by means of an electrodynamic vibration generator (shaker), supplied by a constant output voltage amplifier. It may also be inferred from the Fourier components of transient force and velocity generated at the point of excitation by a hammer. In the case of continuous excitation by a shaker, neither the force nor the resulting velocity autospectra are uniform (independent of frequency) because of interaction between the shaker and the excited structure. However, experience shows that, in practice, the force spectrum generated by a shaker is much more uniform than the velocity spectrum, and the input power spectrum exhibits peaks at the resonance frequencies of the structure at which $\mathrm{Re}\,\{\tilde{Y}(\omega)\} \gg \mathrm{Im}\,\{\tilde{Y}(\omega)\}$ and $|\tilde{Y}(\omega)|^2 \approx [\mathrm{Re}\,\{\tilde{Y}(\omega)\}]^2$. Hence, the dominant contributions to the spectra of both power and velocity are associated with peaks in the $\mathrm{Re}\,\{\tilde{Y}(\omega)\}$, and equation (5c) may be written approximately as

$$\int_{\Delta\omega} P_{in}(\omega)\,\mathrm{d}\omega \approx \left[\int_{\Delta\omega} S_{vv}(\omega)\,\mathrm{d}\omega\right]\left[(1/\Delta\omega)\int_{\Delta\omega} \mathrm{Re}\,\{\tilde{Y}(\omega)\}\,\mathrm{d}\omega\right]^{-1}. \tag{6}$$

The frequency-average value of the real part of the mobility of a *spatially uniform* structure of total mass M is given by $\langle \mathrm{Re}\,\{\tilde{Y}(\omega)\}\rangle_\omega = \pi n(\omega)/2M$, where $n(\omega)$ is the corresponding frequency-average modal density, provided that the averaging band includes a number of modal resonance frequencies [5].

Hence, for such structures equation (6) may be written as

$$(P_{in})_{\Delta\omega} \approx (\overline{v^2}_{in})_{\Delta\omega} 2M/\pi n(\omega). \tag{7}$$

In the application of the power injection method, a number of 'randomly distributed' excitation points is selected. On the basis of the hypothesis that equation (7) holds for non-uniform structures in terms of an (unknown) effective mass, one can write

$$\langle P_{in}\rangle_x \approx \langle \overline{v^2}_{in}\rangle_x 2m_e/\pi n, \tag{8}$$

in which the explicit frequency dependence has been dropped, $\langle\ \rangle_x$ indicates 'space average' and an effective mass which is related to the choice of excitation points replaces the actual mass of the substructure.

The SEA power balance equations (1) may now be written as

$$\begin{bmatrix} \sum_{i\neq 1} M_{1i} + M_i & -M_{12} & \cdot & -M_{1k} \\ -M_{12} & \sum_{i\neq 2} M_{2i} + M_2 & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ -M_{1k} & \cdot & \cdot & \sum_{i\neq k} M_{ki} + M_k \end{bmatrix} \begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ \cdot \\ \cdot \\ \cdot \\ E_k/n_k \end{bmatrix}$$

$$= (2/\pi) \begin{bmatrix} \langle v_{e1}^2 \rangle m_{e1}/n_1 \\ \langle v_{e2}^2 \rangle m_{e2}/n_2 \\ \cdot \\ \cdot \\ \cdot \\ \langle v_{ek}^2 \rangle m_{ek}/n_k \end{bmatrix}, \quad (9)$$

in which $\langle v_e^2 \rangle$ in the average mean square velocity at the excitation points.

The subsystem energies may be written as $E = m_e \langle v^2 \rangle$, where $\langle v^2 \rangle$ is the measured space-average mean square response velocity and m_e is the effective mass associated with the response measurements (which may not be the same as that associated with the power input points). On the assumption, however, that these two effective masses may be equated, one obtains:

$$\begin{bmatrix} \sum_{i\neq 1} M_{1i} + M_1 & -(m_{e2}/m_{e1})(n_1/n_2)M_{12} & -(m_{ek}/m_{e1})(n_1/n_k)M_{1k} \\ -(m_{e1}/m_{e2})(n_2/n_1)M_{12} & \sum_{i\neq 2} M_{2i} + M_2 & \cdot \\ \cdot & \cdot & \cdot \\ -(m_{e1}/m_{ek})(n_k/n_1)M_{1k} & \cdot & \sum_{i\neq k} M_{ik} + M_1 \end{bmatrix}$$

$$\times \begin{bmatrix} \langle v_1^2 \rangle \\ \langle v_2^2 \rangle \\ \cdot \\ \langle v_k^2 \rangle \end{bmatrix} = (2/\pi) \begin{bmatrix} \langle ve_1^2 \rangle \\ \langle ve_2^2 \rangle \\ \cdot \\ \langle ve_k^2 \rangle \end{bmatrix}. \quad (10)$$

These equations contain two sets of directly measured response autospectra and require no measurement of input power, which is one of the major sources of error in SEA experiments.

The elements of the M matrix can be determined in the usual manner by injecting power sequentially into each subsystem and, in each case, measuring space-average input and response velocities on all subsystems. Each power transfer coefficient M_{ij} can be obtained from the product of the values of the two off-diagonal elements in which it appears. The power dissipation coefficients M_i can then be obtained from the values of the diagonal elements of equation (10). No estimates of subsystem effective masses or modal densities are required: the former is a common source of uncertainty in experimental determination of SEA parameters. The results for a two-subsystem model are presented explicitly in the Appendix.

This formulation has one practical disadvantage compared with the measurement of input force and response velocity, namely that velocities at excitation and response points do not satisfy the principle of vibrational reciprocity. Hence, this method requires twice as many measurements as that based upon transfer mobility measurement. On the other hand, the equations in terms of response velocities are generally better conditioned than those in terms of total energies, because power flow is proportional to difference between modal energies, not total energies.

4. CONCLUSION

The concept of coupling loss factor, as employed in SEA, is criticized as being dependent upon physical quantities which do not directly influence the transmission of vibrational power from one subsystem to another. An alternative concept, the 'power transfer coefficient', which takes a form similar to the modal overlap factor, is proposed. It is suggested how this quantity may be determined by applying a variant of the conventional power injection method. The proposed method obviates the need to measure input power or to estimate subsystem effective masses and modal densities.

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APPENDIX: TWO-SUBSYSTEM MODEL

The following approximate solutions of equations (10) for a two-subsystem model are based upon the assumption that $\langle v_2^2\rangle_2/\langle v_1^2\rangle_2 \gg \langle v_2^2\rangle_1/\langle v_1^2\rangle_1$, which will be the case except when the system is close to a state of equipartition modal energy (very strong coupling):

$$M_{12} \approx K^{-1}[(\langle v_1^2\rangle_2/\langle v_2^2\rangle_2)/(\langle ve_1^2\rangle/\langle v_1^2\rangle_1)], \tag{A1}$$

$$K \approx [(\langle v_1^2 \rangle_2 / \langle ve_2^2 \rangle)/(\langle v_2^2 \rangle_1 / \langle ve_1^2 \rangle)]^{1/2} = (me_2/me_1)(n_1/n_2), \tag{A2}$$

$$M_1 \approx (2/\pi)(\langle ve_1^2 \rangle / \langle v_1^2 \rangle_1) - M_{12}, \qquad M_2 \approx (2/\pi)(\langle ve_2^2 \rangle / \langle v_2^2 \rangle_2) - M_{12}. \tag{A3, A4}$$

Here $\langle v_i^2 \rangle_j$ indicates the spatial-average mean square response velocity of subsystem i when external excitation is applied to subsystem j.